EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

Welcome to Grade 8! In the first topic of Module 1, students will be learning about operations (mathematical processes such as addition and subtraction) with terms that have **exponents.** They will learn how to use definitions and properties, often referred to as the laws of exponents, to perform these operations. Students will start by investigating the properties of exponents using only positive exponents (e.g., 8^2 or $(-7)^4$), and then they will extend their knowledge to exponents of zero (e.g., 8^0) and **negative exponents** (e.g., 5^{-2} or $(-3)^{-4}$).

You can expect to see homework that asks your child to do the following:

- Write a **repeated multiplication representation** using exponents.
- Recognize when standard numbers are showing an exponential pattern. For example, 2, 4, 8, 16, and 32 are equal to 2^1, 2^2, 2^3, 2^4, and 2^5, respectively.
- Change a given number to an **exponential expression** with a given **base**. For example, 25 to 5^2.
- Determine whether an exponential expression is positive or negative.
- Simplify expressions using the properties/laws of exponents, including the **zeroth power** and negative powers.
- Explain his work, and prove that two expressions are equivalent by referencing the definition or property/law used.

SAMPLE PROBLEM *(From Lesson 6)*

$(5^{-3})^4 = \left(\frac{1}{5^3}\right)^4$ By definition of negative exponents

$= \left(\frac{1}{5^3}\right) \times \left(\frac{1}{5^3}\right) \times \left(\frac{1}{5^3}\right) \times \left(\frac{1}{5^3}\right)$ By definition of exponential notation

$= \frac{1}{5^{3+3+3+3}}$ By 1st law of exponents

$= \frac{1}{5^{12}}$

$= 5^{-12}$ By definition of negative exponents

Properties of Exponents/Laws of Exponents

For any numbers x, y and all integers (0, and positive and negative numbers that are not fractions) a, b, the following rules apply:		
Name of Rule	General Example	Another Example
1st Law of Exponents	$x^a \cdot x^b = x^{a+b}$	$3^6 \times 3^8 = 3^{6+8} = 3^{14}$
2nd Law of Exponents-Power to a Power	$(x^a)^b = x^{ab}$	$((-6)^4)^2 = (-6)^{4\cdot 2} = (-6)^8$
3rd Law of Exponents	$(xy)^a = x^a y^a$	$(5g)^3 = 5^3 \cdot g^3$
Division of Exponents; Consequence of 1st Law for Division	$\frac{x^a}{x^b} = x^{a-b}$	$\frac{x^{10}}{x^2} = x^{10-2} = x^8$
Fraction to a Power; Consequence of 3rd Law for Division	$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$	$\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$
For any positive number x, and all integers b, the following rule applies:		
Definition of Negative Exponents	$x^{-b} = \frac{1}{x^b}$	$5^{-2} = \frac{1}{5^2}$

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are just a few tips to help you get started:

- Study the exponent law or definition your child learned in class each night. Teamwork is powerful!
- Hold a race with your child. Write a variety of numbers that can be written as exponential expressions, like 16, 25, and 27, on index cards and place the cards face down. As you take turns flipping over the cards, race to be the first to convert each number into an exponential expression. For example, 16 to 4^2 or 2^4, 25 to 5^2, 27 to 3^3, 81 to 9^2 or 3^4, and 125 to 5^3.

TERMS

Associative property of multiplication: You can change the grouping of terms being multiplied without changing the resulting value, or product. For example, $3 \cdot (x \cdot y) = (3 \cdot x) \cdot y$.

Base: In the term $3y^6$, the y is the repeating factor, or base, and may be a number or a variable.

Coefficient: A constant factor (not to be confused with a “constant”) in a variable term. For example, in the term $3y^6$, the 3 represents the coefficient, and is multiplied by y^6.

Commutative property of multiplication: You can multiply terms in any order and not change the resulting value, or product. For example, $3 \cdot y = y \cdot 3$.

Exponent: In the term $3y^6$, the 6 is the exponent or power. The exponent tells you how many times to multiply the base (y) by itself.

Exponential expression: A mathematical term with a base, exponent, and sometimes a coefficient. For example, the term $3y^6$ is an exponential expression and it means $3 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$.

Exponential notation: The method used to write a repeated multiplication expression. $\frac{9}{7} \times \frac{9}{7} \times \frac{9}{7} \times \frac{9}{7}$ can be written as $\left(\frac{9}{7}\right)^4$. When your base is a fraction or a negative number, the base should be placed inside parentheses.

Negative exponents: When a base, x, is raised to a negative power, $-y$, it is equivalent to the fraction $\frac{1}{x^y}$. For example, $3^{-2} = \frac{1}{3^2}$.

Ratio: A comparison of the sizes of two values. Ratios are written as $A{:}B$ (e.g., 1:4), or “A to B” (e.g., 1 to 4) where the number A is first and the number B is second.

Value of the ratio: The value of the ratio $A{:}B$ is the quotient $\frac{A}{B}$ as long as B is not zero. For example, the ratio 6:10 has a value of $\frac{6}{10}$ or 0.6.

Zeroth power: Any base raised to the power of zero has a value of 1. For example, $x^0 = 1$, $\left(\frac{4}{7}\right)^0 = 1$, $(-2)^0 = 1$.

MODELS

Repeated Multiplication Representation: $(-2.3)^9 = \underbrace{((-2.3) \times \cdots \times (-2.3))}_{9\ times}$

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

In Topic B, students are introduced to **scientific notation,** which is a convenient way to write numbers that are very large or very small. Students learn to convert standard numbers to scientific notation and perform operations on numbers in many forms. Finally, students compare numbers written in various forms to put them in order or to determine which number has the greatest or least value.

After your child has completed Lesson 11, LEARN MORE by viewing a video called "Powers of Ten," which demonstrates positive and negative powers of 10. Visit: eurmath.link/powers-of-ten.

You can expect to see homework that asks your child to do the following:

- Use the **order of magnitude** of a number to determine the next greatest **power of ten,** and put numbers in order according to their value. The larger the magnitude, the larger the number's value.
- Solve real-life problems using numbers written in scientific notation.
- Convert numbers written in standard form to scientific notation, and vice versa. Represent those numbers on a calculator.
- Determine whether a number represented in scientific notation is very large or very small in value.
- Perform calculations on numbers represented in scientific notation.
- Change a given unit of measure to a different unit of measure.

SAMPLE PROBLEMS *(From Lessons 9 and 10)*

The table below shows the debt of the three most populous states and three least populous states.

State	Debt (in dollars)	Population (2012)
California	407,000,000,000	38,000,000
New York	337,000,000,000	19,000,000
Texas	276,000,000,000	26,000,000
North Dakota	4,000,000,000	690,000
Vermont	4,000,000,000	626,000
Wyoming	2,000,000,000	576,000

How much larger is the combined debt of the three most populous states than that of the three least populous states? Express your answer in scientific notation.

$$\begin{aligned}(1.02 \times 10^{12}) - (1 \times 10^{10}) &= (1.02 \times 10^2 \times 10^{10}) - (1 \times 10^{10})\\ &= (102 \times 10^{10}) - (1 \times 10^{10})\\ &= (102 - 1) \times 10^{10}\\ &= 101 \times 10^{10}\\ &= (1.01 \times 10^2) \times 10^{10}\\ &= 1.01 \times 10^{12}\end{aligned}$$

SAMPLE PROBLEMS *(continued)*

Approximately how many times greater is the total population of California, New York, and Texas compared to the total population of North Dakota, Vermont, and Wyoming?

$$\frac{8.3 \times 10^7}{1.892 \times 10^6} = \frac{8.3}{1.892} \times \frac{10^7}{10^6}$$
$$\approx 4.39 \times 10$$
$$\approx 43.9$$

The combined population of California, New York, and Texas is about 43.9 times greater than the combined population of North Dakota, Vermont, and Wyoming.

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are just a few tips to help you get started:

- The idea of "how many times larger" comes up often in this topic. To determine "how many times larger," you need to divide. For example, if the area of your living room is 330 square feet and the area of your bathroom is 110 square feet, you would need to divide 330 by 110 to determine that the living room is 3 times larger than the bathroom. Discuss with your child why "how many times larger" indicates the need to divide. Perform some of these calculations together, gathering ideas from real-life numbers such as sports statistics and merchandise prices.
- When you are in the grocery store, garage, or workroom, discuss with your child the different units of measure you encounter. This will help your child form stronger mental models of what an inch looks like and how many ounces are in a pound, for example. With this practice your child will become better prepared to answer questions about measurement units.

TERMS

Order of magnitude: The exponent of the power of 10 when a decimal is expressed in scientific notation. For example, in scientific notation, the decimal 192.7 is represented as 1.927×10^2, so its order of magnitude is 2 (the exponent in 10^2).

Power of ten: A term with the number 10 as its base. For example, 10^3 is a power of 10 that equals 1,000.

Product: The answer to a multiplication problem.

Product of a decimal: The result of multiplying any number and a decimal.

Scientific notation: The representation of a very large or very small number as the product of a decimal and a power of 10. The decimal must have a value greater than or equal to 1 and less than 10. For example, 2.41×10^5 is in scientific notation, while 24.1×10^4 is not because the decimal value, 24.1, is greater than 10. Scientific notation is used when the number is too big or too small to be conveniently written in standard form.

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

This module is all about geometry. Until now, students may have thought of two objects as being congruent if they were the same shape and the same size. In Topic A, we will lay the groundwork for arriving at a more precise mathematical definition of congruence. Students will be doing hands-on work as they **transform** (slide, turn, or flip) points, segments**,** lines and shapes.

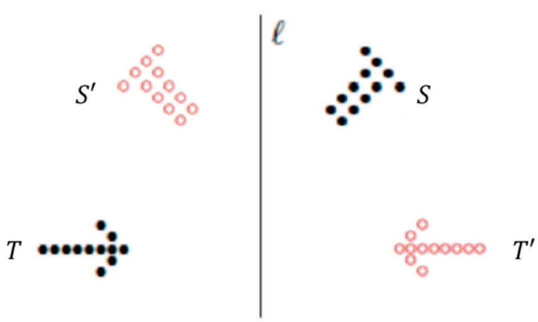
To LEARN MORE about transformations, visit:
eurmath.link/translation, eurmath.link/reflection, eurmath.link/rotatecw, and eurmath.link/rotateccw.
The videos were developed by Sunil Koswatta.

You can expect to see homework that asks your child to do the following:

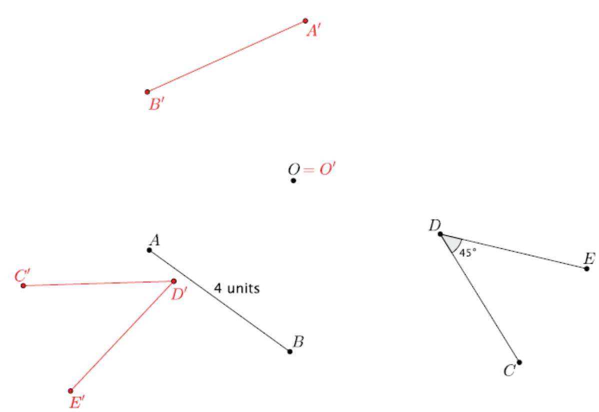
- Identify transformations (**translation, rotation, reflection**) that have been performed on shapes.
- Translate (slide), rotate (turn), and reflect (flip) objects using given criteria.
- Use accurate labeling and precise language when performing transformations.
- Determine lengths of segments and measures of angles (e.g., 45°, 90°) after a transformation has been performed.
- Understand the special consequences of rotations of 180°.

SAMPLE PROBLEMS *(From Lessons 4 and 5)*

The original images are in black, and the reflected (flipped) images are in red.



Let $\overline{AB}$ be a segment of length 4 units and $\angle CDE$ be 45°. Let there be a rotation by d degrees, where $d < 0$, about O. Find the images of the given figures.



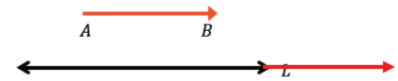
Verify that students have rotated around center O in the clockwise direction.

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

TERMS

Basic rigid motion: Any transformation (such as a flip or a turn) in which the distance between any two points is kept the same. There are three basic types of rigid motions: translations (slides), reflections (flips), and rotations (turns).

Coincide: If you translate line L along a vector $\overrightarrow{AB}$, the new part of the line, in red, just extends the original line. We say line L and its image coincide.



Collinear: Points that are on the same line.

Image: An object that has been turned, flipped, or slid to a new location. This image should have a label with a prime (see below) to distinguish it from the original object.

Length notation: As a shortcut to writing, "The length of the segment AB is," students use the notation $|AB|$.

Map: When an object maps onto another object, that means they're congruent, or exactly the same. We say that Object 1 maps to, or maps onto, Object 2.

Preserving: Maintaining the original measure. For example, an angle preserves its measure when rotated, so a 45° angle will still be 45° after it has been rotated.

Prime notation: Original objects, shapes, or points are labeled with capital letters. When a point, shape, or object, P, is transformed, a prime is added to its label, P'. If that image is then transformed again, it will be labeled with two primes, P''. This continues for each new transformation.

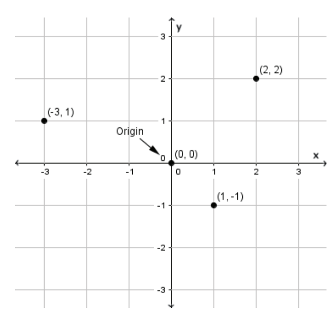
Reflect/Reflection: A type of transformation that moves every point in the original object across a line of reflection (a line directly in the middle between the original and the new image). This is often referred to as a flip over a line. When describing a reflection, a student should write, "The original object was reflected over (or across) $\overleftrightarrow{AB}$." In the first sample problem, the line of reflection is ℓ.

Rotate/Rotation: A type of transformation that turns an object around a point. When describing a rotation, a student should write, "The original object was rotated around point P by 45°." Rotations going in a clockwise direction have negative degree measures, while rotations in a counterclockwise direction have positive degree measures.

Transformation: The movement of a point, segment, line, or object. There are four transformations in Grade 8: translation (slide), rotation (turn), reflection (flip), and dilation (stretch or shrink).

Translate/Translation: A type of transformation that moves every point in the original object along a vector to a new location. This is often referred to as a slide along a vector. When describing a translation, a student should write, "The original object was translated along vector $\overrightarrow{AB}$."

Vector: A line segment that has a direction; it is represented by a symbol on which one end is a point and the other end is an arrow. Its notation is $\overrightarrow{AB}$ which means that when you translate a shape, you will start at point A and move the shape along the vector, stopping at point B.



Coordinate: The location of a point on the coordinate plane, written (x, y). The first number is always the x-value of the point (left/right), and the second number is always the y-value of the point (up/down).

Origin: The point where the two axes intersect in the coordinate plane. Its coordinates are (0, 0).

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

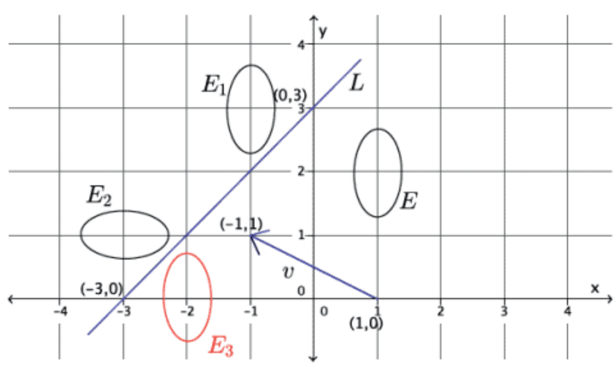
Now that students have learned to perform single transformations, they will begin sequencing transformations, or performing more than one type of transformation on the same shape. Students will investigate to determine whether performing multiple transformations changes the properties—measurements, for instance—of a shape that stayed the same during a single transformation. Precise language is essential in Topic B because students must accurately explain which object is being transformed and what each transformation requires.

You can expect to see homework that asks your child to do the following:

- Using given criteria, perform the appropriate sequence of rigid motions (translate, rotate, and reflect) on objects.
- Use accurate labeling and precise language when performing a **sequence of transformations**.
- Determine lengths of segments and measures of angles after a sequence of transformations has been performed.
- Determine whether the order in which a sequence of transformations is performed will affect the final location of the image.

SAMPLE PROBLEM *(From Lesson 10)*

This image shows a sequence of transformations performed on Object E.



To create Object E_1, translate (slide) Object E along the vector from point $(1, 0)$ on the coordinate plane/grid to point $(-1, 1)$. (NOTE: Students can also explain this translation on the coordinate plane as 1 up and 2 left.)

To create Object E_2, rotate (turn) Object E_1 around point $(-1, 1)$ $90°$. Notice that the rotation is in the counterclockwise direction.

To create Object E_3, reflect (flip) Object E_2 across line L.

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

Many of the activities that you and your child worked on during Topic A will still be useful in Topic B. Here are some other ideas for you to help your child at home.

- If your child is having trouble identifying each type of transformation, try demonstrating the three types with refrigerator magnets. Discuss what happened to the corners and/or edges of the magnet after each transformation. Are they in a new location? For example, suppose you labeled the upper left corner point A and the upper right corner point B. Is point A still to the left of point B after the reflection? Considering these questions may help your child understand that you have literally flipped the object and that the labels on the points of a flipped image will not be in the same order as on the original. Have these same conversations about translations and rotations as well.
- Sentence starters are a great way to help your child understand exactly which language to use for each transformation. For now, provide your child with a copy of each sentence exactly as it appears below. Have your child fill in the blanks. As your child begins to master the language, remove the hints that appear in parentheses, and have your child once again fill in the blanks. For example, you would now provide the starter *Translate* ___ *using* ___. Eventually, your child should be able to complete each sentence with only the first word as a starter (e.g., *Translate* ___) and then without starters.

  Translate ___ (object name) using ___ (vector name).
  Reflect ___ (object name) across the line ___ (line name).
  Rotate ___ (object name) around point ___ (point name) ___ (number) degrees.

TERMS

Sequence of transformations: A set of transformations performed in a particular order (e.g., a translation followed by a rotation).

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

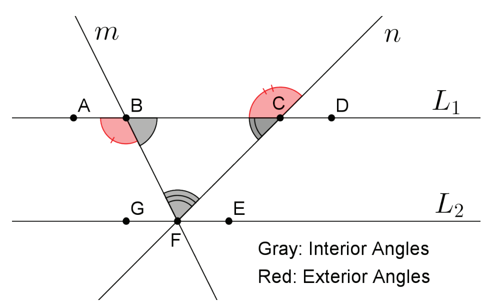
In Topic C, students discover and apply a precise definition for **congruence**. They examine the angles formed when a **transversal** crosses **parallel lines**; they also examine the angles inside and outside of a triangle. To pull all of these relationships together, students begin examining diagrams in which two or more transversals cross parallel lines, creating triangles.

You can expect to see homework that asks your child to do the following:

- Use sequences of transformations to determine whether two figures are congruent.
- Use precise language to describe the congruence by describing the sequence of transformations that was performed.
- Determine the relationships between angles and missing angle measurements in a diagram in which parallel lines are cut by a transversal. Describe these relationships using precise language with transformations.
- Determine the measures of missing angles in diagrams with triangles.
- Determine whether two lines are parallel given the measure of the angles in the diagram.

SAMPLE PROBLEM *(From Lesson 13)*

The figure below shows parallel lines L_1 and L_2. Let m and n be transversals that intersect L_1 at points B and C, respectively, and L_2 at point F, as shown. Let A be a point on L_1 to the left of B, D be a point on L_1 to the right of C, G be a point on L_2 to the left of F, and E be a point on L_2 to the right of F.



a. Name a triangle in the figure. ***$\triangle BCF$***

b. Name a straight angle that will be useful in proving that the sum of the measures of the interior angles of the triangle is 180°. ***$\angle GFE$***

c. Our goal is to show that the sum of the measures of the interior angles of the triangle is equal to the measure of the straight angle. Show that the measures of the interior angles of a triangle have a sum of 180°. Write your proof below.

The straight angle $\angle GFE$ comprises $\angle GFB$, $\angle BFC$, and $\angle EFC$.
Alternate interior angles of parallel lines are equal in measure.
For that reason, $m\angle BCF = m\angle EFC$ and $m\angle CBF = m\angle GFB$.
Since $\angle GFE$ is a straight angle, its measure is equal to 180°.
Then, $m\angle GFE = m\angle GFB + m\angle BFC + m\angle EFC = 180^\circ$.
By substitution, $m\angle GFE = m\angle CBF + m\angle BFC + m\angle BCF = 180^\circ$.
Therefore, the sum of the measures of the interior angles of a triangle is 180° (angle sum of triangles).

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Review the topic vocabulary with your child. An Internet search for *vocabulary review games* will generate many fun options. Using index cards or small pieces of paper to make flashcards could also be helpful.
- Some students may become distracted or confused when there are multiple angles in a diagram. Here are two ways you can help.
    - Use sticky notes to cover the parts of the diagram that your child is not currently using so he can focus on the angles in front of him.
    - Use colored pencils to outline the angles named in the diagram. After connecting the vertices (dots) referenced in the name of the angle, you may want to shade the inside of the angle so your child can see the entire angle.
- Help your child draw upon knowledge and skills from prior topics in this module. Encourage her to use the transparency from Topic A to outline the angle she is working with in the current diagram. Guide her to use the basic rigid motions (translate, rotate, reflect) to map the angle onto another angle in the diagram. Then, work with your child to determine the appropriate word (congruent, **corresponding**, **alternate interior**, etc.) to describe the relationship between the two angles.

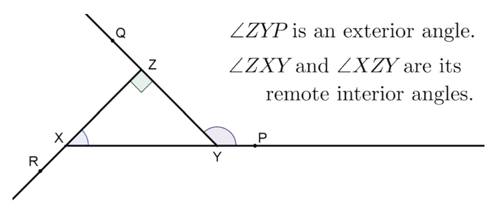
TERMS

Congruent/Congruence: Objects are congruent if one object can be mapped onto (fit exactly on top of) the other after a sequence of transformations has been performed. $\triangle ABC \cong \triangle A'B'C'$ is read as, "Triangle ABC is congruent to Triangle A prime B prime C prime."

Exterior angle: An angle formed when one side of a triangle is extended.

Remote interior angles: The two angles inside the triangle that do not touch the exterior angle.

Triangle angle sum: The measures of three angles of any triangle add up to 180 degrees.



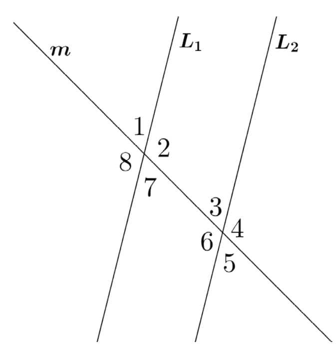
Parallel lines: Two lines that will never touch. If line W is parallel to line Y, we can write $W \parallel Y$.

Corresponding angles: Two angles that are on the same side of the transversal in corresponding positions (e.g., angles 2 and 4 in the picture).

Alternate interior angles: Two angles on different sides of the transversal and between the parallel lines (e.g., angles 2 and 6 in the picture).

Alternate exterior angles: Two angles on different sides of the transversal and outside the parallel lines (e.g., angles 4 and 8 in the picture).

Supplementary angles: Two angles whose measures add up to 180 degrees (e.g., angles 1 and 8 in the picture).



Transversal: Any line that intersects two or more (usually parallel) lines. In the picture, line m is the transversal.

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

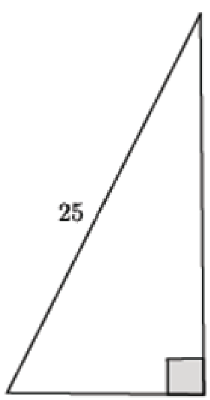
In Topic D, students are introduced to the **Pythagorean theorem**, $a^2 + b^2 = c^2$, a rule about right triangles. Students will perform the basic rigid motions and apply what they have learned about congruence to prove the Pythagorean theorem (i.e., to verify it). After proving the theorem, students will use it to find the length of one side of a right triangle, given the lengths of the other two sides.

You can expect to see homework that asks your child to apply the Pythagorean theorem to do the following:

- Determine the missing length of the **hypotenuse of a right triangle**.
- Determine the missing length of a **leg of a right triangle**.
- Determine the lengths of segments in a graph as well as in real-life situations (e.g., the length of a ladder leaning against a wall that creates a right triangle with the wall and the floor).

SAMPLE PROBLEMS *(From Lesson 16)*

Use the Pythagorean theorem to find the missing length of the leg in the right triangle.

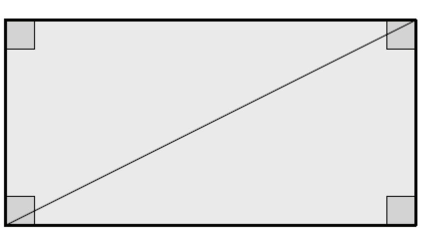


Let b represent the missing leg length.

$$15^2 + b^2 = 25^2$$
$$15^2 - 15^2 + b^2 = 25^2 - 15^2$$
$$b^2 = 625 - 225$$
$$b^2 = 400$$
$$b = 20$$

The length of the leg is 20 units.

Given a rectangle with dimensions 5 cm and 10 cm, as shown, find the length of the diagonal, if possible.



Let c represent the length of the diagonal, in centimeters.

$$c^2 = 5^2 + 10^2$$
$$c^2 = 25 + 100$$
$$c^2 = 125$$

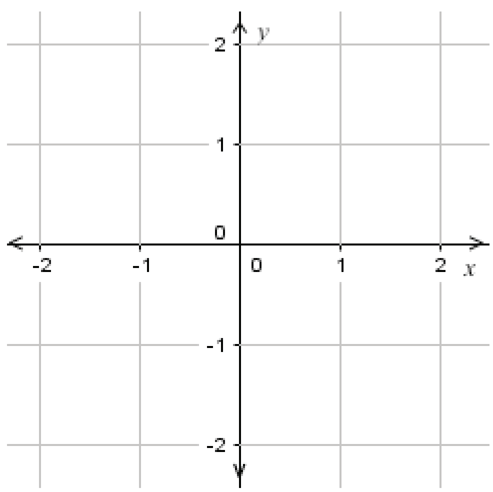
The length of the diagonal in centimeters is the positive number c that satisfies $c^2 = 125$.

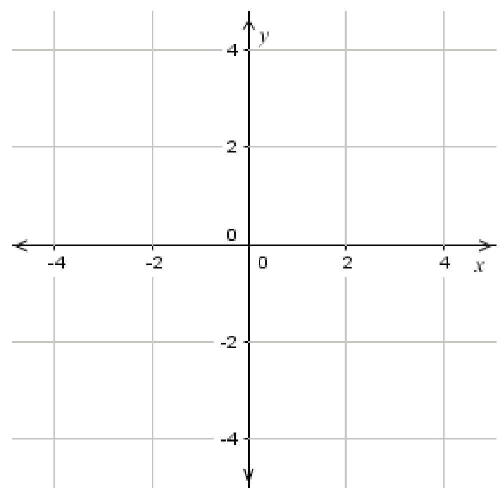
Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Many mathematicians have argued that the Pythagorean theorem is the most useful discovery in mathematics. Here's a way to test the theorem at home. Gather a measuring tape, a pencil, and a piece of string exactly 5 feet long. Determine a place in your home where a right angle should exist, such as where the wall meets the floor. Working with your child, start at the floor and measure 3 feet up the wall. Mark this point lightly with a pencil. Then, starting from the same point on the floor, measure 4 feet along the floor, and make another pencil mark. Stretch the string from the mark on the wall to the mark on the floor, making it the hypotenuse in the triangle you just created. If the wall meets the floor at an exact 90° angle, the distance from mark to mark should be exactly 5 feet; thus, the string should reach exactly from one mark to the other. Have your child describe what type of triangle you just created. It is a 3–4–5 triangle—a right triangle in which the lengths of the sides have a ratio of 3 to 4 to 5. We have worked with these right triangles in class.
- Your child will continue to work with the coordinate plane throughout Grade 8. Continue to practice plotting points using graph paper, a tile floor, or a grid of your own invention. Try assigning different scales to the axes. For example, instead of having each square on the graph paper (or tile on the floor, etc.) represent one unit, have each square represent two, five, or ten units. Challenge your child to graph points based on the new axis scale.





TERMS

Hypotenuse of a right triangle: The longest side of the right triangle; it is opposite the right angle.

Leg of a right triangle: One of the two shorter sides of the right triangle. Together, the legs form the right angle.

Pythagorean theorem: If the triangle is a right triangle, then $leg_1^{\,2} + leg_2^{\,2} = hypotenuse^2$, or $a^2 + b^2 = c^2$.

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

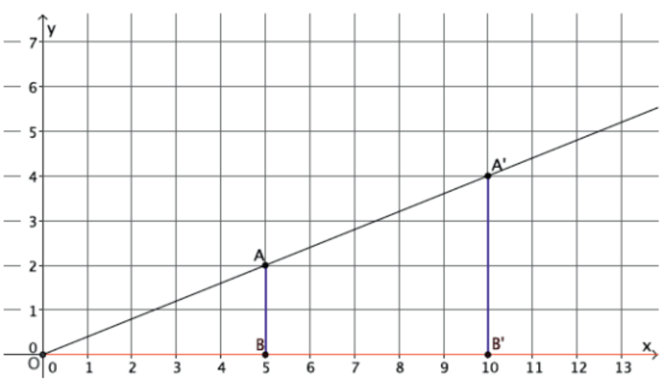
In Module 3, students are introduced to a new transformation called a **dilation**, which results in an image that is the same shape but a different size than the original. Because a dilation **magnifies (enlarges)** or **shrinks (reduces)** the original shape, it is not a rigid motion. Students will use a rule called the **fundamental theorem of similarity**, or FTS, to examine the effect of dilations on **coordinates**. Through this work, students will develop a precise **definition of dilation**. During this module, your child will be asked to use a ruler, a compass, and a calculator. Making these tools available at home will help your child complete his work.

You can expect to see homework that asks your child to do the following:

- Use side lengths to calculate the **scale factor** of a dilation and classify it as an enlargement or a reduction.
- Use the definition of dilation to solve for the length of an unknown side in the original shape or dilated shape, as well as calculate the scale factor used in the dilation.
- Create images by dilating an original figure using the given **center of dilation** and scale factor. Students will dilate objects with straight or curved sides.
- Determine the sequence of transformations used on an original object to create an image.
- Calculate the scale factor used to return an image back to the original figure.
- Use the fundamental theorem of similarity to solve for segment lengths and coordinates of points and to find congruent angles and parallel lines.

SAMPLE PROBLEMS *(From Lesson 5)*

1. Find the length of segment $A'B'$ using the diagram. Explain.



If the points A and B are both dilated by a scale factor of 2, the segments AB and $A'B'$ will be parallel, with the length of segment $A'B'$ (4) equal to twice the length of segment AB (2). Therefore, $|A'B'| = 2|AB|$, and $4 = 2 \cdot 2$.

2. Point D $(0, 11)$ is dilated from the origin by scale factor $r = 4$. What are the coordinates of point D'?

$$D'(4 \cdot 0, 4 \cdot 11) = D'(0, 44)$$

3. Point $E(-2, -5)$ is dilated from the origin by scale factor $r = \frac{3}{2}$. What are the coordinates of point E'?

$$E'\left(\frac{3}{2} \cdot (-2), \frac{3}{2} \cdot (-5)\right) = E'\left(-3, -\frac{15}{2}\right)$$

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Maps are perfect examples of dilations. Actual distances are shrunk to fit on paper or a screen. Talk to your child about how the definition of dilation is used to make accurate maps. Find the scale factor for a map by checking the map key. For example, if 1 inch represents 5 miles, the scale for the actual distance on the map is $\frac{1}{5}$.
- Using the definition of dilation, provide numbers for two of the following three parts of a dilated image: original length, image length, or scale factor. Have your child solve for the unknown part. For example, if the image length is 10 cm and the original length is 2 cm, your child would solve for the scale factor, r, by dividing the image length (10) by the original length (2), which equals 5.

TERMS

Angle-preserving: Maintaining the original measure of an angle (e.g., 45 degrees) when a transformation is performed.

Center of dilation: The point from which the dilation was magnified or shrunk.

Coordinates: The location of a point on the coordinate plane, written as (x, y). The first number is always the x-value of the point (left/right), and the second number is always the y-value of the point (up/down).

Definition of dilation: The precise definition used to solve for an unknown segment length or scale factor. The definition is written as $|A'B'| = r|AB|$, meaning that the length of the new, dilated segment is equal to the scale factor times the length of the original segment.

Dilate/Dilation: A type of transformation that moves every point in the original object closer to or farther from a point, called the center of dilation. Dilations are often referred to as enlargements or reductions. When describing a dilation, a student should write the following: *The original object was dilated by a scale factor of [insert number] about (or using) center point* P.

Effect of dilation on coordinates: When the center of dilation is the origin and the scale factor is r, an original point (x, y) becomes (rx, ry). For example, multiply the original coordinates (2, 5) by a scale factor of 4 to find the new (dilated) coordinates (8, 20).

Fundamental theorem of similarity: If you dilate points A and B from the same center point C using the same scale factor, corresponding side $\overline{DE}$ of the magnified/reduced shape will have the following properties:

a. It will be parallel to side $\overline{AB}$ of the original shape.
b. Its length will be equal to the scale factor times the length of side $\overline{AB}$.

Magnification/Enlargement: A dilation that lengthens each side of the original shape by a given scale factor. The object's image will also be farther from the center of dilation. Every enlargement has a scale factor with a value greater than 1.

Scale factor: A number associated with the size of the dilation. This number can be multiplied by the original lengths to obtain the new lengths. We often use the variable r to represent the scale factor.

Shrinking/Reduction: A dilation that shortens each side of the original shape by the given scale factor. The object's image will also be closer to the center of dilation. Every reduction has a scale factor with a value between, but not equal to, 0 and 1.

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

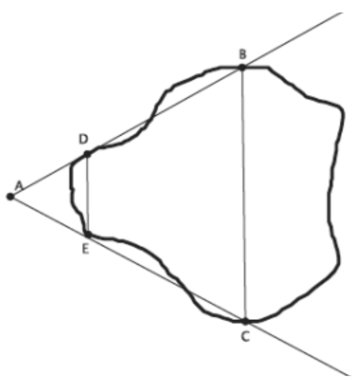
In Topic B, students formally define **similarity** and investigate the properties of similar objects. Looking specifically at triangles, students learn to tell whether two triangles are similar using techniques other than dilation or the basic rigid motions. After determining that two triangles are similar, students use equivalent ratios to find the unknown side lengths of a triangle. Finally, students apply their knowledge of similarity to real-world tasks, such as finding heights of buildings and determining distances that are too large to measure with a typical measuring tool.

You can expect to see homework that asks your child to do the following:

- Describe a dilation followed by a sequence of rigid motions that would map one shape onto another.
- Determine whether two objects are similar using **angle-angle criterion** or **proportional** side relationships.
- Use dilation and rigid motion to prove that similarity is **symmetric** and **transitive**.
- Given that two triangles are similar, solve for the unknown side length.
- Use similar triangle relationships to solve problems with real-world contexts.

SAMPLE PROBLEM *(From Lesson 12)*

A geologist wants to determine the distance across the widest part of a nearby lake. The geologist marked off specific points around the lake so that the line containing segment DE would be parallel to the line containing segment BC. Segment BC is selected specifically because it is the widest part of the lake. Segment DE is selected specifically because it is a short enough distance to measure easily. The geologist sketched the situation, as shown below.



a. Has the geologist done enough so far to use similar triangles to help measure the widest part of the lake? Explain your answer.

Yes, based on the sketch, the geologist found a center of dilation at point A. The geologist marked points around the lake that, when connected, make parallel lines. The triangles are similar by the angle-angle (AA) criterion. Corresponding angles of parallel lines are equal in measure, and the measure of $\angle DAE$ is equal to itself. Since there are two pairs of corresponding angles that are equal, $\triangle DAE \sim \triangle BAC$.

b. The geologist made the following measurements: $|DE| = 5$ feet, $|AE| = 7$ feet, and $|EC| = 15$ feet. Does she have enough information to complete the task? If so, determine the length across the widest part of the lake. If not, state what additional information is needed.

Yes, there is enough information about the similar triangles to determine the distance across the widest part of the lake using the AA criterion.

Let x represent the length of segment BC; then, $\frac{x}{5} = \frac{22}{7}$.

We are looking for the value of x that makes the fractions equivalent. Therefore, $7x = 110$ and $x \approx 15.7$. The distance across the widest part of the lake is approximately 15.7 feet.

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

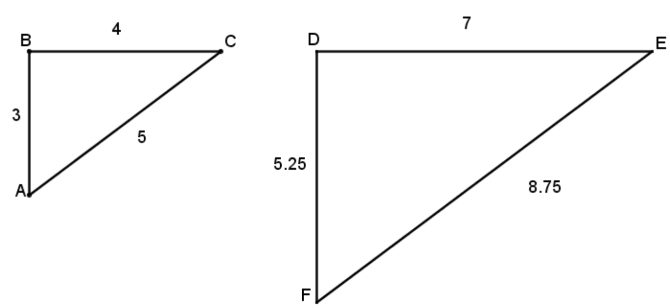
- Similar triangles are often used to solve problems in the real world. Help your child master the information needed to determine whether two triangles are similar. Review facts about angle relationships from Grade 8 Module 2 Topic C, where students explored the angles created when a line (called a transversal) passes through parallel lines. Students should be familiar with corresponding, alternate interior, alternate exterior, and supplementary angles from Module 2.
- Play vocabulary games. Work with your child to create vocabulary and key concept cards using the Terms sections of this and previous Parent Tip Sheets. Write a term or concept on the front of each card and the definition on the back. Then take turns with your child drawing a card, defining the term or concept on the front, and checking the answer on the back. You might compete to be the first to provide five correct answers. You might also use the board from a favorite board game and advance around it by defining terms and concepts correctly.

TERMS

Angle-angle criterion: Two triangles are similar if two angles of one triangle are congruent (equal in measure) to two angles from the other triangle.

Proportional: Two quantities (such as lengths or widths of objects) are proportional when they have the same relative size in relation to each other. Here is an example of the notation you could use to show that the side lengths of two shapes are proportional. If triangle *ABC* is similar to triangle *FDE*, then we can write the following:

$$\frac{|AB|}{|BC|} = \frac{|FD|}{|DE|}$$



Similar/Similarity: Two objects are similar if there is a dilation followed by a sequence of rigid motions that would map one object onto another. Similar shapes preserve angle measures, and the lengths of their sides are proportional. We use the symbol ~ to represent similarity.

Symmetric property: Similar in reasoning to the commutative properties of addition and multiplication, the symmetric property says that if $A \sim B$ (*A* is similar to *B*), then $B \sim A$ (*B* is similar to *A*).

Transitive property of similarity: This property states that one similar figure can be substituted for another similar figure. If $A \sim B$ (*A* is similar to *B*) and $B \sim C$ (*B* is similar to *C*), then $A \sim C$ (*A* is similar to *C*).

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

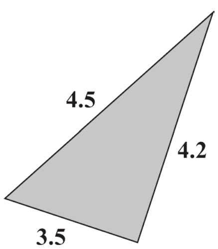
In Topic C, we return to the **Pythagorean theorem**. In this exposure to the theorem, students are presented with a proof that involves similar triangles and the angle-angle criterion. Once again, students apply the Pythagorean theorem to find the measures of unknown side lengths in right triangles.

You can expect to see homework that asks your child to do the following:

- Use the Pythagorean theorem to solve for the measure of an unknown side length in a right triangle.
- Use the properties of **perfect squares** to apply to perfect square decimals. For example, if $c^2 = 121$, then $c = 11$. Likewise, if $c^2 = 1.21$, then $c = 1.1$.
- Determine whether a triangle is a right triangle by using the **converse of the Pythagorean theorem**.

SAMPLE PROBLEM *(From Lesson 14)*

The numbers in the diagram below indicate the lengths of the sides of the triangle. Is the triangle shown below a right triangle? Show your work, and answer in a complete sentence.



If this were a right triangle, the side measuring 4.5 ***would be the longest side and would therefore be the hypotenuse. We need to check whether*** $3.5^2 + 4.2^2 = 4.5^2$ ***is a true statement. The left side of the equation is equal to*** 29.89***. The right side of the equation is equal to*** 20.25***. That means*** $3.5^2 + 4.2^2 = 4.5^2$ ***is not true, so the triangle shown is not a right triangle.***

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

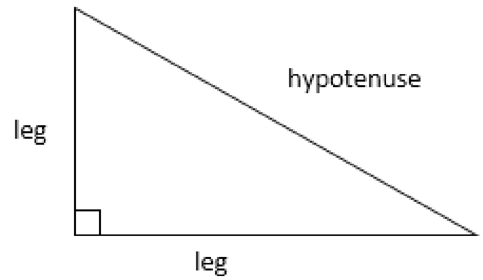
- Do you have a tall tree or a flagpole in your yard? Use the shadow activity found in Lesson 12 Problem Set 1. On a sunny day, position yourself such that the end of your shadow and the end of the tree's shadow match up. Ask your child to measure the following: your distance from the tree, your height, and the length of your shadow. Ensure that the measurements are all in one unit (e.g., 3 feet 6 inches should either be 42 inches or 3.5 feet). Then, challenge your child to use that data with equivalent ratios to determine the height of the tree. Next, your child can use the Pythagorean theorem and the data gathered so far to determine the **hypotenuse** lengths of the two triangles formed by you, the tree, and your shadows.
- Right angles are all around you. Continue to point out right angles (or what appear to be right angles) in your environment. Discuss with your child ways to determine whether an angle is actually a right angle.
- Work with your child to remember the perfect squares. Your child should be able to identify the first fifteen perfect squares and know what number squared results in each perfect square (See Terms).

TERMS

Perfect square: A number that is the result of squaring an integer base. The first fifteen perfect squares are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, and 225.

Hypotenuse of a right triangle: The side of the right triangle that is opposite the right angle. This is also the longest side of the right triangle.

Legs of a right triangle: The two sides of the right triangle that form the right angle.



Pythagorean theorem: If the triangle is a right triangle, then $leg_1{}^2 + leg_2{}^2 = hypotenuse^2$, or $a^2 + b^2 = c^2$.

Converse of the Pythagorean theorem: If $leg_1{}^2 + leg_2{}^2 = hypotenuse^2$, or $a^2 + b^2 = c^2$, then the triangle is a right triangle.

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

In Module 4 Topic A, students begin to make connections between proportional relationships and **linear expressions** and equations. They transcribe the information from word problems into expressions and equations and then evaluate or solve. Students learn that an equation may have one **solution**, no solution, or many solutions.

You can expect to see homework that asks your child to do the following:

- Write statements using symbolic language. For example, twice a number less 4 is transcribed as $2x - 4$, where x represents a number.
- Determine whether an expression or equation is linear or nonlinear.
- Solve linear equations, explain the **properties of equality** used to find the solutions, and check those solutions.
- Write and solve equations to find the measures of angles in triangles.
- Determine whether an equation has a unique (one) solution, no solution, or infinitely many solutions.

SAMPLE PROBLEMS *(From Lessons 7 and 9)*

1. Solve the linear equation $x - 9 = \frac{3}{5}x$. State the property that justifies each of your steps.

The left side of the equation, $x - 9$, and the right side of the equation, $\frac{3}{5}x$, are transformed as much as possible.

$$x - 9 = \frac{3}{5}x$$

$$x - x - 9 = \frac{3}{5}x - x \quad \textit{Subtraction property of equality}$$

$$(1-1)x - 9 = \left(\frac{3}{5} - 1\right)x \quad \textit{Distributive property}$$

$$-9 = -\frac{2}{5}x$$

$$-\frac{5}{2}(-9) = -\frac{5}{2}\left(-\frac{2}{5}x\right) \quad \textit{Multiplicative property of equality}$$

$$\frac{45}{2} = x$$

2. Give a brief explanation as to what kind of solution(s) you expect the following linear equation to have. Transform the equation into a simpler form if necessary.

$$11x - 2x + 15 = 8 + 7 + 9x$$

$$\mathbf{11x - 2x + 15 = 8 + 7 + 9x}$$

$$\mathbf{(11-2)x + 15 = (8+7) + 9x}$$

$$\mathbf{9x + 15 = 15 + 9x}$$

I notice that the coefficients of the x are the same, specifically 9, and that the constants, 15, are also the same. Therefore, this equation has infinitely many solutions.

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Ask your child to transform each side of an equation from class using the **commutative**, **associative**, and/or **distributive** properties. Then have your child solve the *new* equation using the properties of equality.
- Place equations from both Lessons 2 and 3 on index cards. Have your child organize the cards into linear and nonlinear equations.

TERMS

Associative property: The grouping in an addition or multiplication problem may change, but the sum or product will remain the same.

Coefficient: In the term $3y^6$, for example, the 3 represents the coefficient, or the number in front of the base (y). It means that y^6 is being multiplied by 3.

Commutative property: The order of an addition or multiplication problem may change, but the sum or product will remain the same.

Consecutive integers: Consecutive integers are integers that come one after another when counting. For example, –6, –5, –4, and –3 are consecutive integers. Likewise, 4, 6, and 8 are consecutive even integers.

Constant of a linear equation/expression: The number that is being added to the variable term. For example, in the linear equation $3x - 4 = 8 + 6x$, –4 and 8 are the constants in the equation.

Distributive property: Allows the numbers in a multiplication problem to be distributed into partial products (i.e., partial answers). The partial products can then be added together to find the product, or the answer to the original multiplication problem (e.g., $3(x + 7) = (3 \cdot x) + (3 \cdot 7) = 3x + 21$).

Exponent: In the term $3y^6$, the 6 is the exponent. The exponent tells you how many times to multiply the base (y) by itself.

Linear expression: The sum/difference of one or more expressions (e.g., $4x - 5$) that consist of either a number, a variable, or the product of a number and a variable, where the variable is raised to the power of 0 or 1. The expression $4x^3 - 5$ is nonlinear because the variable is raised to the third power.

Properties of equality: Each property of equality states that if you add (subtract, multiply, or divide) by a number on one side of an equation, you can add (subtract, multiply, or divide) by that same number on the other side of the equation without changing the value of the variable or the equality of the statement.

Reciprocal: The number obtained by inverting a fraction. For example, 4 (which is $\frac{4}{1}$) and $\frac{1}{4}$ are reciprocals, as are $\frac{3}{4}$ and $\frac{4}{3}$. When you multiply a number by its reciprocal, the product is always 1.

Solutions of a linear equation: There are three possibilities for the solution to a linear equation. If both sides of the equation are transformed using the commutative, associative, and/or distributive properties and you notice that ...

- the coefficients of the variable terms are the same, and the constants are also the same (e.g., $3x + 4 = 4 + 3x$ in both instances), then the equation has infinitely many solutions.
- the coefficients of the variable terms are the same, but the constants are different (e.g., $-8x + 7 = -8x - 6$ in both instances), then the equation has no solution.
- the coefficients of the variable terms are different regardless of the constant (e.g., $6 - \frac{1}{4}x = 7x + 4$ in both instances), then the equation has one unique solution.

Variable term: In a linear equation, the part of the expression containing the coefficient and variable. For example, in the linear equation $3x - 4 = 8 + 6x$, $3x$ and $6x$ are the variable terms.

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

In Topic B, students write linear equations to represent **constant rate** problems. The lessons in this topic introduce students to the standard form of an equation in two variables and ask students to write, interpret, and graph information from various situations.

You can expect to see homework that asks your child to do the following:

- Write and solve problems with **proportional relationships** involving speed, distance, time, and other constant rates.
- Write a **linear equation in two variables**.
- Given the value of one variable, solve a two-variable linear equation to determine the value of the other variable.
- Compute information for a constant rate problem, or a linear equation, and graph the data in the **coordinate plane**.
- Given data in a coordinate plane, determine whether the data represent a given linear equation.
- Find solutions to an equation, and plot the solutions as points on a coordinate plane.
- Graph linear equations on the coordinate plane.

SAMPLE PROBLEM *(From Lesson 11)*

Juan types at a constant rate. He can type a full page of text in $3\frac{1}{2}$ minutes. How many pages, p, can Juan type in t minutes?

a. Write a linear equation representing the number of pages Juan can type in any given time period.

Let C represent the constant rate that Juan types in pages per minute. Then, $\frac{1}{3.5} = C$ and $\frac{p}{t} = C$; therefore, $\frac{1}{3.5} = \frac{p}{t}$.

$$\frac{1}{3.5} = \frac{p}{t}$$

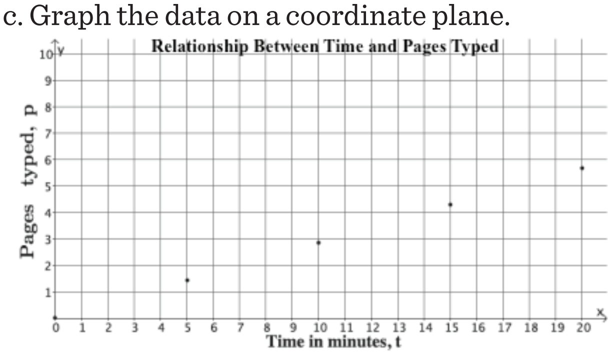
$$(t)\frac{1}{3.5} = \frac{p}{t}(t)$$

$$\frac{1}{3.5}t = p$$

b. Complete the table below. Use a calculator, and round your answers to the tenths place.

t (time in minutes)	Linear Equation: $p = \frac{1}{3.5}t$	p (pages typed)
0	$p = \frac{1}{3.5}(0)$	0
5	$p = \frac{1}{3.5}(5)$	$\frac{5}{3.5} \approx 1.4$
10	$p = \frac{1}{3.5}(10)$	$\frac{10}{3.5} \approx 2.9$
15	$p = \frac{1}{3.5}(15)$	$\frac{15}{3.5} \approx 4.3$
20	$p = \frac{1}{3.5}(20)$	$\frac{20}{3.5} \approx 5.7$

c. Graph the data on a coordinate plane.



Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Point out activities involving rate in everyday life (i.e., things you do that can be measured in terms of the time it takes to do them, such as number of words typed per minute or number of hot dogs sold per hour). Have a conversation about whether those rates are actually constant or whether we simply speak of the average rate as if it were constant. For example, do you actually drive through town at a constant rate of 30 mph, or is that your average rate? We will use constant rate often in this topic to mean average rate.
- Give your child a rate, and have her determine an equivalent rate. For example, if you walk at an average rate of 3 miles per hour, how many hours will it take you to walk 9 miles? Since $\frac{3 \text{ miles}}{1 \text{ hour}} = \frac{9 \text{ miles}}{x \text{ hours}} = \frac{9 \text{ miles}}{3 \text{ hours}}$, it will take you 3 hours. How many miles can you walk in $1\frac{1}{2}$ hours? Since $\frac{3 \text{ miles}}{1 \text{ hour}} = \frac{x \text{ miles}}{1.5 \text{ hours}} = \frac{4.5 \text{ miles}}{1.5 \text{ hours}}$, you can walk 4.5 miles in $1\frac{1}{2}$ hours.
- Write a two-variable equation for the situations described above, making sure to define the variables. For the example above, if m represents the number of miles walked and t represents the number of hours you walk, the two-variable equation is $m = \frac{3}{1}t$, or just $m = 3t$.

TERMS

Constant rate: The rate at which something can be done that is the same over any time interval. For example, a typist might type at a constant rate of 200 words per minute, and a racing cyclist might ride at a constant speed of 25 miles per hour.

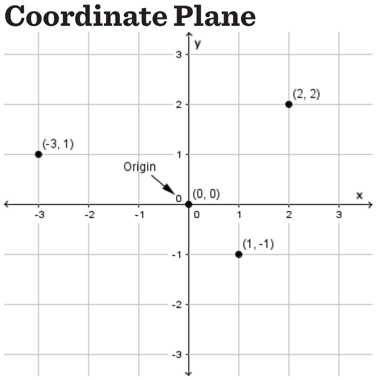
Coordinates: The location of a point on the coordinate plane, written as (x, y). The first number is always the x-value of the point (left/right), and the second number is always the y-value of the point (up/down).

Linear equation in two variables: An equation with two variables (e.g., $y = 2x + 4$). The variables have exponents of 1 or 0 only and cannot be the denominator in a fraction when the equation is in standard form. All linear equations can be graphed as straight lines in the coordinate plane. Other examples include $x = 3$ (implies $0y + 1x = 3$) and $\frac{1}{4}c - 8v = 9$.

Proportional relationship: When two quantities (e.g., the weight of an item and its price) increase or decrease at the same rate, their relationship is proportional. If 1 pound of tomatoes sells for $4 (1:4) and 2 pounds sell for $8 (2:8), the weight and price are proportional. That is, each measure in the second quantity (4 and 8), when divided by its corresponding measure in the first quantity (1 and 2), produces the same number (4), called a constant.

MODELS

Coordinate Plane



EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

Topic C extends students' work with constant rate as it applies to the **slope** of a line. Students determine the slope by using any two points from the graph of a line. Students then apply the slope of the line to the **slope-intercept form** to find the equation of that line. For example, if the slope is 3, the slope-intercept form of the line could be $y = 3x + 8$. Last, students compare various proportional relationships represented in graphs, tables, equations, and descriptions.

You can expect to see homework that asks your child to do the following:

- Determine whether the slope of a line is positive or negative, and then find the exact value of the slope or ***y*-intercept point**. The data used may be given in graphs, tables, equations, or descriptions.
- Confirm that the slope of a line stays the same when using two different points on the line to determine the slope.
- Using the properties of equality, transform an equation from **standard form** to slope-intercept form and vice versa.
- Given points on a line—or the graph, table, equation, or description of the line—determine one or more of the other representations (i.e., points, graph, table, equation, or description) of the line.
- Determine whether two equations result in the same line when graphed.
- Find and graph various solutions to an equation.

SAMPLE PROBLEMS *(From Lesson 22)*

A faucet leaks at a constant rate of 7 gallons per hour. Suppose y gallons leak in x hours. Express the situation as a linear equation in two variables.

$\frac{y}{x} = 7$ ***or*** $y = 7x$

Another faucet leaks at a constant rate, and the table below shows the number of gallons, y, that leak in x hours.

Number of Hours (x)	Number of Gallons (y)
2	13
4	26
7	45.5
10	65

Determine the rate at which the second faucet leaks.

Let m represent the rate at which this faucet leaks in gallons per hour.

$$m = \frac{(26-13)}{(4-2)}$$

$$m = \frac{13}{2}$$

$$m = 6.5$$

The second faucet leaks at a rate of 6.5 gallons per hour.

Which faucet has the worse leak? That is, which faucet leaks more water over a given time interval?

The first faucet has the worse leak because the rate is greater: 7 gallons per hour compared to 6.5 gallons per hour.

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Give your child proportional relationships in different forms (e.g., an equation such as $y=\frac{5}{2}x-6$ and a description such as “Mary types at a rate of $3\frac{1}{3}$ words per minute.”). Challenge him to determine which situation has the greater rate. (Mary has a greater rate.) Equations and constant rate descriptions like these can be found in many of the lessons in this topic.
- Use class examples to find the slope of a situation presented in different forms. Give your child two points, a table of points, a graph, an equation, or a description of a situation, and ask her to find the slope of the line that represents the situation.
- Write a two-variable equation, and ask your child to transform the equation such that it says “$y=$” or “$p=$” or that one of the variables equals the rest. You can find these examples in the lessons, or you can make up equations yourself. For example, $2x+3y=-6$ would transform to $y=-\frac{2}{3}x-2$ when rewriting the equation in slope-intercept form, or “$y=$”.

TERMS

Intercept point: The point $(0, b)$ at which a line intersects the y-axis where b is the y-value of the y-intercept point. There is also an x-intercept point, $(x, 0)$, where the line intersects the x-axis.

Slope: A number that describes the steepness or slant of a line. The unit rate (e.g., number of miles per hour) or rate of change (how one quantity changes in relation to another) is often interpreted as the slope of a graph. Lines that go up from left to right have a positive slope, and lines that go down from left to right have a negative slope. The slope, m, of a line can be found using the following equation:

$$m=\frac{\text{difference in } y\text{-values}}{\text{difference in } x\text{-values}}=\frac{p_2-r_2}{p_1-r_1}=\frac{y_2-y_1}{x_2-x_1}.$$

Slope-intercept form of a linear equation: A linear equation written as $y=mx+b$, where m represents the slope of the line and b represents the y-value of the y-intercept point.

Standard form of a linear equation: The standard form of a linear equation is written as $ax+by=c$ (e.g., $2x+3y=17$).

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

In Topic D, students continue their work with linear equations by exploring **simultaneous equations (systems of equations)** using graphs, as well as multiple algebraic methods. Students discover that, as with linear equations in one variable, a system can have a unique solution, no solution, or infinitely many solutions. Topic E extends systems of equations to an application of the **Pythagorean theorem**.

You can expect to see homework that asks your child to do the following:

- Write a system of equations for situations involving constant rate.
- Graph a system of equations and interpret the point where the lines intersect as the solution to the system.
- Substitute numbers for specific variables to verify the solution for simultaneous equations.
- Determine whether a system has a unique solution, no solution, or infinitely many solutions.
- Solve simultaneous equations by using the computational methods of elimination and substitution. (See Sample Problem.)
- Apply techniques for solving systems of equations to real-life situations, including finding **Pythagorean triples**.

SAMPLE PROBLEM *(From Lesson 28)*

Determine the solution to the system of equations by eliminating one of the variables. Verify the solution using the graph of the system.

$$\begin{cases} x - 4y = 7 \\ 5x + 9y = 6 \end{cases}$$

Transform one of the equations to create inverses that will cancel or eliminate one of the variables.

$$-5(x - 4y) = -5(7)$$
$$-5x + 20y = -35$$

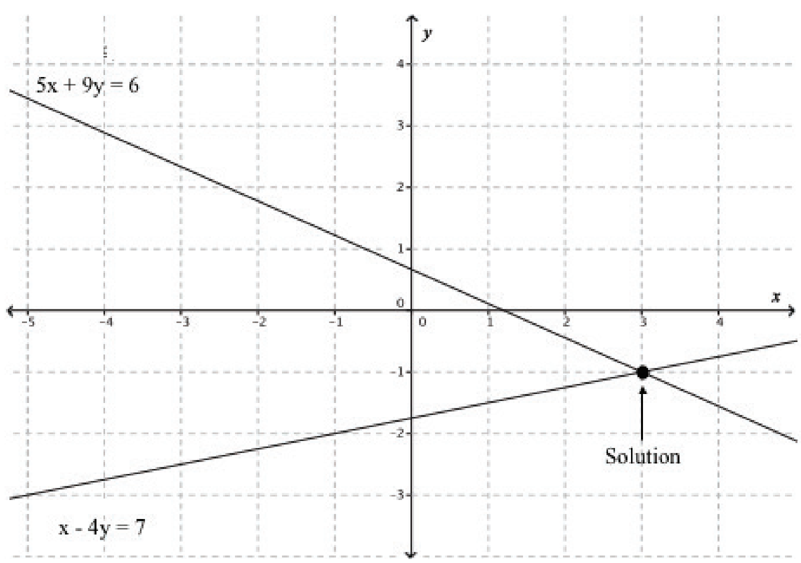
Now there is a* new *system where one of the variables will eliminate.

$$\begin{cases} -5x + 20y = -35 \\ 5x + 9y = 6 \end{cases}$$

$$-5x + 20y + 5x + 9y = -35 + 6$$
$$29y = -29$$
$$y = -1$$

$$x - 4y = 7$$
$$x - 4(-1) = 7$$
$$x + 4 = 7$$
$$x = 3$$

***The solution is* (3, −1).**



Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

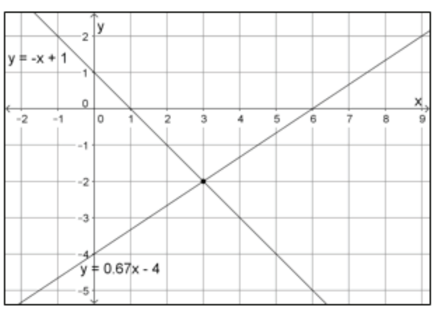
You can help at home in many ways. Here are some tips to help you get started.

- Help your child practice identifying additive inverses so she becomes more comfortable using them in the elimination method for solving systems of equations. For example, the additive inverse of $3x$ is $-3x$, and the additive inverse of $-\frac{1}{4}y$ is $\frac{1}{4}y$.
- Continue to work with your child on transforming equations from standard form to slope-intercept form. Give your child any equation written as $ax + by = c$, and have him rewrite that equation in the form "$y =$." For example, give your child the equation $3x - 2y = 10$. After your child transforms the equation to $y = \frac{3}{2}x - 5$, ask him to identify the slope, $\frac{3}{2}$, and the y-intercept point, $(0, -5)$.

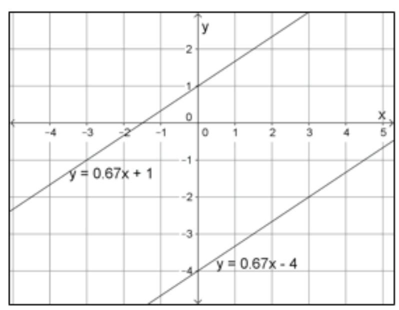
TERMS

Ordered pair: Two numbers written in a fixed order, usually as (x, y).

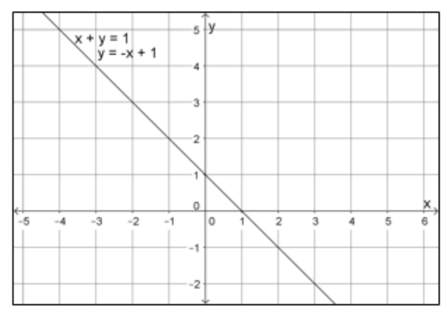
Simultaneous equations/Systems of equations: Two or more two-variable equations that have one common solution, graphically represented by where the graphs intersect. There are also systems of equations with no solution, which would graph as parallel lines, and some with infinitely many solutions, which would graph as the same line. (See images below.)



One Solution



No Solution



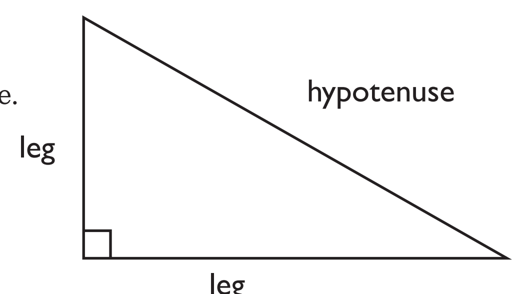
Infinite Solutions

Hypotenuse of a right triangle: The longest side of the right triangle. The hypotenuse is opposite the right angle.

Leg of a right triangle: One of the two shorter sides of the right triangle. Together, the legs form the right angle.

Pythagorean theorem: If the triangle is a right triangle, then $leg_1^2 + leg_2^2 = hypotenuse^2$, or $a^2 + b^2 = c^2$.

Pythagorean triple: Three positive integers that represent the lengths of the sides of a right triangle and that successfully fulfill the Pythagorean theorem.



EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

In this topic, students learn the concept of a **function**, its formal definition, and how it works as an input–output machine. For example, if the function is *multiply by 5*, the output will always equal the input times 5. Students learn that the equation $y = mx + b$ defines a **linear function** whose graph is a straight line, and that a **nonlinear function** is a set of ordered pairs that graph as something other than a straight line. Students begin comparing two functions represented in different ways. For example, students are presented with an equation, a word problem, the **graph of a function**, and the **table of values** that represent a function and are asked to determine which function has the greatest **rate of change**.

You can expect to see homework that asks your child to do the following:

- Interpret the graph of a function to identify key features, including whether the function is linear or nonlinear.
- Find the **average rate of change**.
- Determine whether a given representation represents a function, and create representations of real-world functions. For example, the water flowing from a faucet into a bathtub is a linear function with relation to time if the flow of water is constant.
- Create a rule (an equation) that represents a function.
- Identify whether a function is **discrete** or **not discrete**.
- Determine **restrictions on the variables**.
- Compare functions and determine which has the greater rate of change.

SAMPLE PROBLEMS *(From Lesson 5)*

The distance that Giselle runs is a function of the amount of time she spends running. Giselle runs 3 miles in 21 minutes. Assume she runs at a constant rate.

a. Write an equation in two variables that represents the distance she ran, y, as a function of the time she spent running, x.

$$\frac{3}{21} = \frac{y}{x}$$

$$y = \frac{1}{7}x$$

b. Use the equation you wrote in part (a) to determine how many miles Giselle can run in 28 minutes.

$$y = \frac{1}{7}(28)$$

$$y = 4$$

Giselle can run 4 miles in 28 minutes.

c. Is the function discrete?

The function is not discrete because we can find the distance Giselle runs for any given amount of time she spends running (e.g., 10.2 minutes).

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

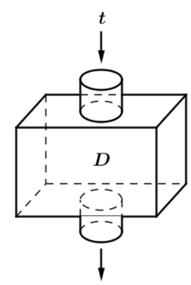
- Help your child understand the restrictions on a variable by discussing real-world situations around you. For example, determine whether the value of a variable can be a fraction or a negative number if it represents a number of people (whole numbers only), a number of ounces (zero or positive numbers only), or a temperature (zero or positive or negative numbers—including fractions—are acceptable).
- Identify the average rate of change in a real-world situation. For example, if you walked 2 miles to the store in 20 minutes, you can determine the average rate by dividing the distance you traveled by the time it took to get there ($\frac{2}{20}$, or 0.1, miles per minute). Take this a step further by asking your child to create a function rule, or equation, that represents the situation. He might say, "For our walk, let m represent the miles walked and t represent the minutes it took to walk that distance; the function rule is $m = 0.1t$."

TERMS

Average rate of change: The average change of one quantity in relation to a second quantity. For example, we rarely walk at a constant rate—we stop at crosswalks, speed up to cross the street, etc.—but we can calculate the average rate of change for a trip by dividing the total distance walked by the time it took to complete the trip.

Discrete: The input (usually the x-value) is restricted to certain values such as integers or whole numbers. For example, if the input is *number of people,* the function is discrete because people can only be represented by whole numbers.

Function: An assignment of exactly one output for each and every input. In the image, the input is t seconds, and the function, D, manipulates t in some way (often according to an equation) and outputs a distance traveled after t seconds.



Graph of a function: The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. The set represents the solution set of the function. For example, in the simple function *multiply by 5*, the set of ordered pairs to graph would include (–2, –10), (–1, –5), (0, 0), (1, 5), (2, 10), (3, 15), and so on.

Linear function: A set of ordered pairs that can be represented by the equation $y = mx + b$ and graphs as a straight line.

Nonlinear function: A set of ordered pairs that graphs as something other than a straight line.

Not discrete: A function in which the input (usually the x-value) can be any value, including fractions, decimals, and negative numbers. For example, a function with temperature as its input is not discrete because temperatures can have positive, negative, and decimal values (e.g., $42.5°$ or $-6°$).

Rate of change: The rate at which one quantity (e.g., distance traveled) changes in relation to another quantity (e.g., time spent traveling). The rate of change of a linear function is the slope of the graph of a line. In most real-world situations, we identify the average rate of change.

Restrictions on a variable: Some functions represent real-world situations and have restrictions on which numbers or types of numbers they can represent. For example, a variable that represents a number of people cannot be a fraction or a negative number.

MODELS

Table of Values

Bags of Candy (x)	1	2	3	4	5	6	7	8
Cost in Dollars (y)	1.25	2.50	3.75	5.00	6.25	7.50	8.75	10.00

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

In this topic, students begin to think of **volume** as the area of two or more two-dimensional shapes stacked on one another. They develop the general **formulas for the volume** of cones, cylinders, and spheres. Students explore how cones, cylinders, and spheres are related and discover that the volume of a cone is one-third the volume of a cylinder with the same dimensions. They also discover that the volume of a sphere is two-thirds the volume of the cylinder that fits tightly around it. Students use these new formulas to solve real-world and mathematical problems related to volume. Please note that in this topic the terms *cylinder* and *cone* generally refer to a right circular cylinder and a right circular cone.

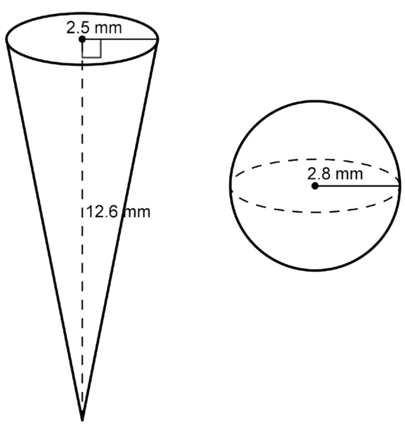
To LEARN MORE by viewing videos about comparing volumes, visit eurmath.link/volume-sphere and eurmath.link/volume-cone.

You can expect to see homework that asks your child to do the following:

- Find the area of two-dimensional figures, including those composed of many shapes.
- Find the volume of three-dimensional figures, including those composed of many **solids**.
- Determine how many of one solid it will take to fill another.
- Identify which solid has a greater volume.

SAMPLE PROBLEMS *(From Lesson 11)*

Use the diagram to answer the problems.



a. Predict which of the figures has the greater volume. Explain.

Student answers will vary. Students will probably say the cone has a greater volume because it looks larger.

b. Find the volume of each figure and determine which has the greater volume.

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi (2.5)^2 (12.6)$$

$$V = 26.25\pi$$

The volume of the cone is** 26.25π **mm^3.

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi (2.8)^3$$

$$V = 29.2\overline{69}\pi$$

The volume of the sphere is about** 29.27π **mm^3.

The volume of the sphere is greater than the volume of the cone.

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Help your child locate solids in your home. For example, a soup can is a perfect representation of a cylinder; dice and children's alphabet blocks are examples of cubes; and a child's pointy party hat is an example of a cone. Measure these objects and calculate the volume of each.
- Have your child explain why the volume formulas for cones and spheres contain fractional values when compared with the volume formula for cylinders with the same radius sizes. The formula for the volume of a cone includes the value *one-third* because it takes three cones to fill a cylinder with the same **base** size. The formula for the volume of a sphere includes the value *four-thirds* because the height of the cylinder is two radii, and when the contents of a sphere are poured into a cylinder with the same radius, the cylinder is two-thirds full. Many videos online show these experiments.

TERMS

Base/Base shape: The two-dimensional shape that is stacked upon itself to create the three-dimensional object. For example, a stack of circles forms a cylinder.

Solids: Three-dimensional figures such as cylinders, cones, rectangular prisms (boxes), and spheres.

Volume: The amount of space inside a three-dimensional object such as a cone or sphere. Volume is measured in cubic units.

Volume formulas: The general formula is $V = Bh$ where V represents the volume of the solid, B represents the area of the base shape, and h represents the height of the solid. Since the base is a circle in the following solids, $B = \pi r^2$, where r is the radius of the circle.

The equation for finding the volume of a cylinder is $V = \pi r^2 h$.

The equation for finding the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

The equation for finding the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

In this topic, students continue to investigate **functions** by connecting a context (word problem) to a set of ordered pairs that model the function at certain inputs. These ordered pairs are then organized in tables and graphs that visually represent the functions. Students discover the relationship between **slope** and rate of change as well as between the y-intercept point and the **initial value** of linear functions. Further investigation leads to determining whether a function represents an **increasing**, **decreasing**, or **constant** relationship. Students close this topic by using graphs and verbal descriptions to explore nonlinear functions.

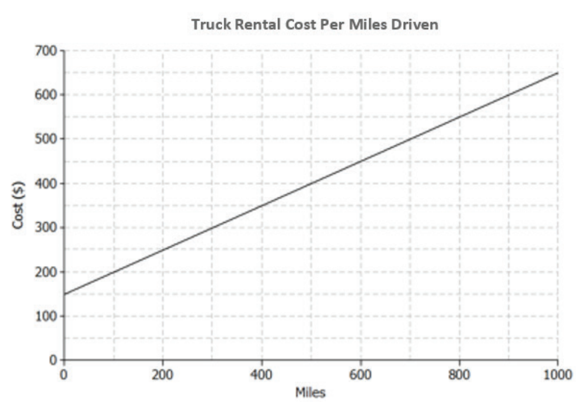
To LEARN MORE by viewing a video about graphing functions, visit eurmath.link/graph-functions.

You can expect to see homework that asks your child to do the following:

- Construct or interpret a table of values, a graph, or an equation that models a linear function.
- Interpret the meaning of values from equations, tables, or graphs in the context of a verbal description.
- Identify which function has a faster rate, steeper slope, or better value.
- Determine the **rate of change** and initial value of a function based on a variety of representations.
- Determine whether a function represents an increasing, decreasing, or constant relationship.
- Explore nonlinear functions by using graphs and verbal descriptions.

SAMPLE PROBLEM *(From Lesson 3)*

Based on the verbal description, create a table, a graph, and an equation.

<table>
<tr><th>Verbal Description</th><th>Table of Values</th><th>Graph</th><th>Equation</th></tr>
<tr>
<td>A truck rental company charges a $150 rental fee in addition to a charge of $0.50 per mile driven. Allow C to represent the total cost of the rental in dollars and m the number of miles driven.</td>
<td>
<table>
<tr><th>Miles Driven</th><th>Total Cost, in dollars</th></tr>
<tr><td>0</td><td>150</td></tr>
<tr><td>200</td><td>250</td></tr>
<tr><td>400</td><td>350</td></tr>
<tr><td>600</td><td>450</td></tr>
<tr><td>800</td><td>550</td></tr>
<tr><td>1000</td><td>650</td></tr>
</table>
</td>
<td>
</td>
<td>$C = 150 + 0.50m$</td>
</tr>
</table>

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

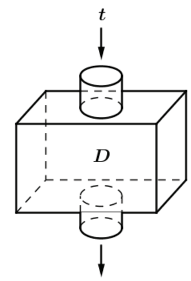
HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Describe situations that can be represented by an increasing, decreasing, or constant linear function. Have your child determine which type of function represents each situation. For example, slowing your car or withdrawing money from your bank account at a constant rate can both be represented by a decreasing linear function. Speeding up or depositing money at a constant rate can both be represented by an increasing linear function. If your car is not moving, or there has been no change in your bank account, a constant linear function represents the situation.
- Use your child's lesson materials to find examples of linear functions represented as tables of values, graphs, verbal descriptions, and equations. Put each representation on a separate index card. Shuffle the cards, and have your child organize them into stacks according to the linear function each card models.

TERMS

Function: An assignment of exactly one output for each and every input. In the image on the right, the input is t seconds, and the function, D, manipulates t in some way (often according to an equation) and outputs a distance traveled after t seconds. For example, if the input, t, represents 50 seconds, and the function multiplies t by 2 meters per second, then the output would be 100 meters.



Initial value: The starting value in a context, often represented by the y-intercept of a graph, which is the value of y where a line intersects the y-axis.

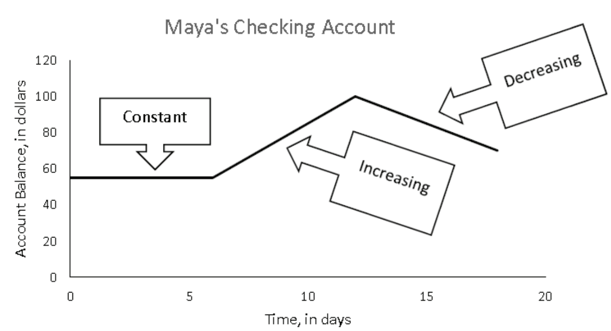
Rate of change: How one quantity changes in relation to another (e.g., miles per hour or price per pound). This is often represented as the slope of a line.

Slope: A number that describes the steepness or slant of a line.

Constant linear function: A function whose graph has a zero slope. It appears as a horizontal line. (See image at right.)

Decreasing linear function: A function whose graph has a negative slope. It appears as a line that tilts downward from left to right across the graph. (See image at right.)

Increasing linear function: A function whose graph has a positive slope. It appears as a line that tilts upward from left to right across the graph. (See image at right.)



EUREKA MATH™ TIPS FOR PARENTS

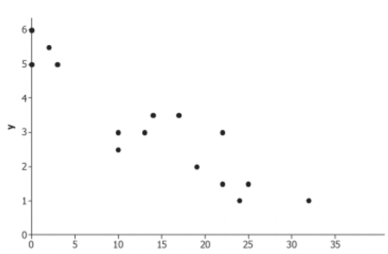
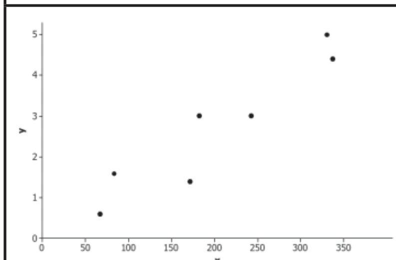
KEY CONCEPT OVERVIEW

In this topic, students connect their study of linear functions to applications involving **bivariate data sets**. A key tool in developing this connection is a **scatter plot**. Students construct scatter plots and focus on identifying linear versus **nonlinear relationships**. Students describe **trends** in the scatter plot, including linear **association**, **clusters**, and **outliers**. Students informally (i.e., without extreme precision) draw a straight line that best represents the data in a scatter plot.

You can expect to see homework that asks your child to do the following:

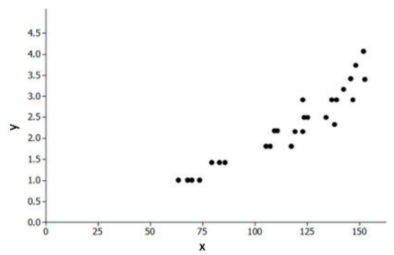
- Construct and interpret a scatter plot and determine the statistical relationship (e.g., **increasing** or **decreasing**) of the data.
- Identify clusters and outliers in a scatter plot.
- Draw a straight line that fits the data in a scatter plot and use it to make predictions about the data.
- Find the equation of the line that fits the data in a scatter plot.
- Match the equation of a line with the scatter plot that best represents that line.

SAMPLE PROBLEMS *(From Lesson 7)*

	Is there a relationship between the two variables used to make the scatter plot? If so, explain the relationship.	If there is a relationship, does it appear to be linear or nonlinear?	If the relationship appears to be linear, is the relationship a **positive linear relationship** or a **negative linear relationship**?
	***Yes, as the value of x increases, the value of y decreases.***	***Linear***	***Negative linear relationship***
	***Yes, as the value of x increases, the value of y increases.***	***Linear***	***Positive linear relationship***

(table continued on next page)

SAMPLE PROBLEMS *(continued)*

	***Yes, as the value of x increases, the value of y increases.***	***Not linear***	***Does not apply.***
	***There is no statistical relationship between price and quality rating.***	***Does not apply.***	***Does not apply.***

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

TERMS

Association: The relationship or trend of a set of data. For example, a data set can be said to have a positive or negative linear association.

Bivariate data set: A set that contains observations about two variables. For example, you can collect data on the weight of a car and on the car's fuel efficiency.

Cluster: A cloud or group of points in a scatter plot.

Decreasing relationship: A relationship in which the y-value decreases as the x-value increases. Data points drop as we move from left to right on the graph.

Increasing relationship: A relationship in which the y-value increases as the x-value increases. Data points rise as we move from left to right on the graph.

Negative linear relationship: A decreasing relationship in which a line with a negative slope represents the data.

No statistical relationship: When the data do not present any pattern, they have no statistical relationship.

Nonlinear relationship: A relationship in which data present as a curve and not a straight line.

Outlier: An unusual point in a scatter plot that does not seem to fit the general pattern.

Positive linear relationship: An increasing relationship in which a line with a positive slope represents the data.

Scatter plot: A graph of the ordered pairs in a data set. (See Sample Problems.)

Trend: A pattern in the data. For example, heavier cars tend to equate to lower fuel efficiency.

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

In this topic, students interpret and use **linear models** to provide explanations for how one variable changes in relation to the other variable for linear and nonlinear associations. Students use scatter plots to describe patterns of positive and negative associations. Students also use graphs and the patterns of linear associations to answer questions about the relationship of the data, including finding the equation of the line that best fits the data.

You can expect to see homework that asks your child to do the following:

- Using descriptive words, write a linear model describing the relationship between two variables. (See Sample Problems.)
- Write an equation, in symbols, that models a given context.
- Interpret the slope and y-intercept of an equation within the context of a problem. Draw a scatter plot and line that best fit the data given.
- Determine whether scattered data are best fit with the graph of a line or a curve. Draw the line or curve to model the data.

SAMPLE PROBLEMS *(From Lesson 10)*

A cell phone company offers the following basic cell phone plan to its customers: A customer pays a monthly fee of $40.00. In addition, the customer pays $0.15 per text message sent from the cell phone. There is no limit to the number of text messages per month that the customer can send, and there is no charge for receiving text messages.

1. Use descriptive words to write a linear model describing the relationship between the number of text messages sent and the total monthly cost.

***Total monthly cost* = $40.00 + *(number of text messages)* · $0.15**

2. Let x represent the **independent variable** and y represent the **dependent variable**. Use these variables to write the function representing the relationship you described in Problem 1.

$y = 40 + 0.15x$ ***or*** $y = 0.15x + 40$

3. During a typical month, Abbey sends 25 text messages. What is her total cost for a typical month?

***Abbey's typical monthly cost is* $40.00 + 0.15(25)*, or* $43.75.**

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Go through the examples and exercises from these lessons with your child. Given the context of each problem, ask your child to interpret the slope and y-intercept of the equation in words, determine the independent and dependent variables, and choose any point that satisfies the equation and describe that point in words. For example, using the equation from the Sample Problems, the slope is the cost, in dollars, per text message; the y-intercept is the monthly fee, in dollars; the independent variable is the number of text messages sent; the dependent variable is the total monthly bill, in dollars; and the point (25, 43.75) means that if you send 25 text messages, your bill will be $43.75.
- Discuss the differences between linear models and nonlinear models. Determine how looking at an equation or a graph can yield enough information to help you decide whether the situation is linear or nonlinear. (Linear models appear as straight lines on a graph and typically have an equation in the form $y = mx + b$. Nonlinear models appear as curves on a graph, and their equations cannot be put in the form $y = mx + b$.)

TERMS

Dependent variable: The variable that represents the output or outcome. Its value changes according to the value of the independent variable.

Independent variable: The variable that represents the input or cause. Its value stands alone and is not affected by any other variables being measured.

Linear model: A way of representing data that follows a linear pattern, using words, graphs, or equations. For example, a linear model might be represented with descriptive words: (an exam score) = 57 + 8(study time). Equivalently, a linear equation that uses symbols might describe the same model: $y = 57 + 8x$, where y represents the exam score and x represents the study time in hours. Thus, an increase of one hour of study time produces an increase of 8 points on the predicted exam score.

Linear pattern: A pattern that can be satisfactorily represented by a line and shows a constant rate of change.

Nonlinear pattern: A pattern that cannot be satisfactorily represented by a line (the data are best represented as a curve) because there is not a constant rate of change.

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

This topic extends the concept of a relationship between variables to **bivariate categorical data**. Students organize bivariate categorical data in a two-way table. They calculate **row** and **column relative frequencies**, decide whether there is an **association** by examining the differences (or similarities), and interpret the frequencies in the context of problems. Students discover that when two categorical variables have an association, knowing the value of one variable can help them predict the value of the other variable.

You can expect to see homework that asks your child to do the following:

- Organize data in a two-way table.
- Calculate relative frequencies.
- Determine whether there is an association in the data.

SAMPLE PROBLEMS *(From Lesson 14)*

Below is a two-way table of row relative frequencies for preferred movie types based on gender.

	Movie Preference			
	Action	**Drama**	**Science Fiction**	**Comedy**
Female	0.25	0.325	0.033	0.392
Male	0.625	0.0125	0.2	0.1625

1. If you randomly select a female participant, would you predict that her favorite type of movie is action? If not, what would you predict and why?

I would not predict that a female participant's favorite type of movie is action. I would predict that a female participant is more likely to prefer comedy since it has the greatest row relative frequency in the female row.

2. Is there an association between the variables of gender and movie preference? Explain your answer.

Yes, there is an association because the row relative frequencies are not the same in each row in the table. If I know a participant's gender, I can use the highest row relative frequency to predict that participant's movie preference.

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Make a two-way table for your child that includes only **frequency** values. (An online search for two-way tables produces many options.) Have her use the data to calculate the **cell relative frequencies**. Using the same original data, she then calculates the row relative frequencies and the column relative frequencies. (See Terms for descriptions on determining these values.) Have a discussion about patterns she sees in the data and whether she thinks there is an association between the variables.
- Guide your child to practice converting fractions to decimals or **percents**. For example, given the fraction $\frac{15}{37}$, have him divide the numerator by the denominator to convert to a decimal that is approximately 0.41. Now he can convert the decimal into a percent: 0.41 = 41%. Other examples include the following: $\frac{5}{8} = 0.625 = 62.5\%$, and $\frac{7}{12}$ is approximately 0.583, or 58.3%.

TERMS

Association: A relationship between the two variables of a bivariate data set. If there is an association between two categories (such as gender and movie preference), knowing a characteristic of one category allows one to make a prediction about the other category. (See Sample Problems.)

Bivariate categorical data: Data on two different categories, such as age and favorite social media site.

Categorical data: Data that can be divided into groups, such as race, gender, age, or favorite ice cream flavor.

Frequency: The number of times the data category occurs in a data set. For example, if 13 students choose chocolate ice cream as their favorite, the frequency is 13, and that number is placed in the cell for chocolate in a two-way frequency table.

Percent: One part in every hundred. One out of 100 is written as $\frac{1}{100}$ and 1%. Percentages can be used to describe relative frequency. For example, if 13 out of 25 students prefer chocolate ice cream, then $\frac{13}{25}$, or 52%, of the students prefer chocolate ice cream.

Proportion: A comparison of a part with the whole. Relative frequency is an example of a proportion.

Relative frequency: A fraction, decimal, or percent that represents how often a specific outcome occurs. To calculate the relative frequency, divide the target outcome by the total number of possible outcomes. For example, if a team won 9 out of 15 games last season, then the relative frequency of winning was $\frac{9}{15}$, 0.6, or 60%.

Cell relative frequency: A cell frequency divided by the table total (i.e., the total number of observations).

Column relative frequency: A cell frequency divided by the column total.

Row relative frequency: A cell frequency divided by the row total.

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

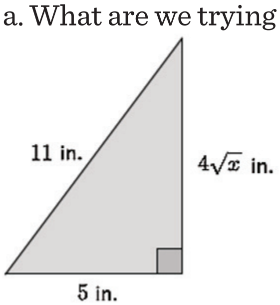
Welcome to the last module of Grade 8. In this topic, students are introduced to **irrational numbers**. As they continue using the Pythagorean theorem to determine side lengths of right triangles, students learn about **square roots** and about irrational numbers in general. Students have previously applied the Pythagorean theorem by using **perfect squares**. Now they learn to estimate the length of an unknown side of a right triangle by determining between which two perfect squares a squared number falls. This leads to the introduction of the notation and meaning of square roots. Students then solve simple equations that require them to find the square root or **cube root** of a number. They then solve multi-step equations by using the properties of equality to transform an equation until it is in the form $x^2 = p$ or $x^3 = p$, where they can use square roots or cube roots to calculate an answer.

You can expect to see homework that asks your child to do the following:

- Determine the length of one side of a right triangle by using the Pythagorean theorem.
- Determine which two **integers** a square root is between and to which integer it is closest.
- Estimate square roots and place them on a number line.
- Solve and check solutions to equations that can be converted to the form $x^2 = p$ or $x^3 = p$.

SAMPLE PROBLEM *(From Lesson 5)*

a. What are we trying to determine in the diagram?



Student work may vary.

We need to determine the value of x so that its square root, when multiplied by 4, satisfies the equation $5^2 + \left(4\sqrt{x}\right)^2 = 11^2$.

b. Determine the value of x. Then check your answer.

$$5^2 + (4\sqrt{x})^2 = 11^2$$
$$25 + 4^2(\sqrt{x})^2 = 121$$
$$25 + 16x = 121$$
$$25 - 25 + 16x = 121 - 25$$
$$16x = 96$$
$$\left(\frac{1}{16}\right)16x = \left(\frac{1}{16}\right)96$$
$$x = 6$$

The value of x is 6.

Check:

$$5^2 + (4\sqrt{x})^2 = 11^2$$
$$5^2 + (4\sqrt{6})^2 = 11^2$$
$$25 + 16(6) = 121$$
$$25 + 96 = 121$$
$$121 = 121$$

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Have your child review the laws of integers from Module 1. There are many instances in which students will be asked to square a term, such as $(8x)^2$. The third law of exponents says that $(8x)^2 = 8^2(x^2) = 64x^2$.
- There are many applications of square roots and cube roots in geometry besides the Pythagorean theorem. Play a memory game to review area and volume formulas with your child. Draw a shape on an index card. On another card, write its area or volume formula. (An online search for *area and volume formula sheet* will produce images and formulas to use.) Place all shape cards facedown in one row and all formula cards facedown in another row. Flip over one card from each row. If the formula card shows the correct formula for calculating the area or volume of the chosen shape, you keep the cards; if not, turn the cards facedown again. Play continues until all matches have been made. The person who collects the most pairs wins.

TERMS

Cube root: The symbol $\sqrt[3]{b}$ denotes the cube root of b. This expression represents a number whose cube is equal to b (e.g., $\sqrt[3]{125} = \sqrt[3]{5^3} = 5$). Not all numbers beneath the cube root symbol are perfect cubes, which means some solutions are not integers and may need to be estimated (e.g., $\sqrt[3]{249} \approx 6.3$).

Infinite decimal: A number with a decimal expansion that never ends. Infinite decimals are usually indicated by a line over the repeating block (e.g., $0.6\overline{75}$) or by an ellipsis following the number (e.g., 0.479456...).

Integer: The whole numbers and their opposites, including zero. The set of integers is $\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$.

Irrational number: A number with a decimal expansion that is infinite and that does not have a repeating block of numbers.

Negative exponents: When a base, x, is raised to a negative power, $-y$, it is equivalent to the fraction $\frac{1}{x^y}$ (e.g., $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$).

Perfect square: A number that is the result of squaring an integer. The first fifteen perfect squares are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, and 225.

Square root: The symbol $\sqrt{b}$ automatically denotes a positive number called the positive square root of b. This expression represents a positive number whose square is equal to b (e.g., $\sqrt{64} = \sqrt{8^2} = 8$). Not all numbers beneath the square root symbol are perfect squares, which means some solutions are not integers and may need to be estimated (e.g., $\sqrt{42} \approx 6.5$).

EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

In this topic, students learn that every number has a **decimal expansion** that is either **finite** (ending) or infinite (never-ending). Students learn many strategies for writing a fraction as a decimal and vice versa. Students then work with infinite decimals such as $0.33333\ldots$ and $0.78146925\ldots$. This work prepares students for understanding how to approximate an irrational number. Students realize that irrational numbers are different from **rational numbers** because irrational numbers have infinite decimal expansions that do not have a repeating block of numbers. Therefore, the value of an irrational number can only be estimated; it cannot be stated exactly. Students then use a number line to compare the estimated value of an irrational number with a rational number in the form of a fraction, decimal, perfect square, or perfect cube. Finally, students approximate pi (π), the most famous irrational number, by using the area of a quarter circle that is drawn on grid paper. Students use this approximation to determine the approximate values of expressions involving π.

You can expect to see homework that asks your child to do the following:

- Convert fractions to finite or infinite decimals and vice versa.
- Write the **expanded form of a decimal** by using powers of 10.
- Approximate an infinite decimal, identify the size of error in the approximation, and find the decimal's location on a number line.
- Determine whether a number is rational or irrational.
- Approximate irrational square roots and cube roots.
- Order a group of rational and irrational numbers and graph those values (or approximations) on a number line.
- Approximate π and use that approximation in calculations.

SAMPLE PROBLEM *(From Lesson 11)*

Between which two consecutive hundredths does $\sqrt{14}$ lie? Show your work.

The number $\sqrt{14}$ lies between the consecutive hundredths of 3.74 and 3.75.

To begin, the number $\sqrt{14}$ is between 3 and 4 because $3^2 < \left(\sqrt{14}\right)^2 < 4^2$. I then checked the tenths intervals between 3 and 4. Since $\left(\sqrt{14}\right)^2$ is closer to 4^2, I began with 3.9 to 4.0. The number $\sqrt{14}$ is not between those values, so I moved to 3.8 and 3.9. That interval did not work either. The number $\sqrt{14}$ is actually between 3.7 and 3.8 because $3.7^2 = 13.69$ and $3.8^2 = 14.44$. Then I looked at the hundredths intervals between 3.7 and 3.8. Since $\left(\sqrt{14}\right)^2$ is closer to 3.7^2, I began with the interval 3.70 to 3.71. It is not between those values; $\sqrt{14}$ is actually between 3.74 and 3.75 because $3.74^2 = 13.9876$ and $3.75^2 = 14.0625$.

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Draw a vertical line down the center of a piece of paper to make a chart. Label the left side of the chart *Rational Numbers* and the right side *Irrational Numbers*. Give your child a number. You can find examples of rational and irrational numbers in the lessons in this topic. Have him identify the number as rational or irrational and write it in the correct half of the chart.
- Teaching is the best indication of learning. Your child will learn many new techniques for finding the decimal expansions of numbers. Have her teach you a few of these techniques. The techniques include using number lines, long division, powers of ten, and expanded form.

TERMS

Decimal expansion: A number written in decimal form. For example, the decimal expansion of $\frac{3}{4}$ is 0.75.

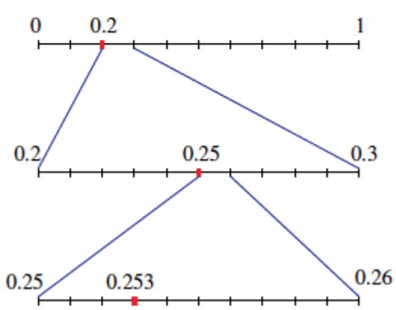
Expanded form of a decimal: The value of a number written as a sum. For example, 0.125 is written as $\frac{1}{10}+\frac{2}{10^2}+\frac{5}{10^3}$ in expanded form.

Finite decimal: A number with a decimal expansion that ends.

Rational number: A number with a decimal expansion that either ends or is infinite with a repeating block.

MODELS

Decimal Placement on a Number Line



EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

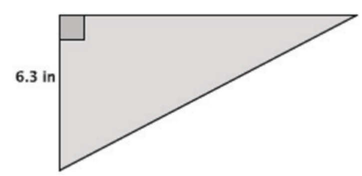
Building on students' new knowledge of square roots, this topic introduces another proof of the Pythagorean theorem (if a triangle is a right triangle, then $a^2 + b^2 = c^2$) and its converse (if $a^2 + b^2 = c^2$, then the triangle is a right triangle). Students learn to determine the approximate length of a side of a right triangle, even when the length is not a whole number. Students also practice explaining proofs in their own words, and they apply the converse of the Pythagorean theorem to make informal arguments about whether certain triangles are right triangles. To finish the topic, students focus on applications of the Pythagorean theorem. They calculate the distance between two points on a diagonal line in the coordinate plane and apply the theorem to a variety of other mathematical and real-world scenarios.

You can expect to see homework that asks your child to do the following:

- Use similar triangles to illustrate the Pythagorean theorem in particular situations.
- Use the Pythagorean theorem to find the unknown length of a side of a right triangle.
- Use the converse of the Pythagorean theorem to determine whether a triangle is a right triangle.
- Find the distance between two points on the coordinate plane.
- Use the Pythagorean theorem in a variety of mathematical and real-world scenarios.

SAMPLE PROBLEM *(From Lesson 18)*

The area of the right triangle shown is 26.46 in^2. What is the perimeter of the right triangle? Round your answer to the tenths place.



Let b inches represent the length of the base of the triangle.

Let h inches represent the height of the triangle, where $h = 6.3$.

$$A = \frac{bh}{2}$$

$$26.46 = \frac{6.3b}{2}$$

$$(2)26.46 = (2)\frac{6.3b}{2}$$

$$52.92 = 6.3b$$

$$\frac{52.92}{6.3} = \frac{6.3b}{6.3}$$

$$8.4 = b$$

Let c inches represent the length of the hypotenuse.

$$6.3^2 + 8.4^2 = c^2$$

$$39.69 + 70.56 = c^2$$

$$110.25 = c^2$$

$$\sqrt{110.25} = \sqrt{c^2}$$

$$\sqrt{110.25} = c$$

The number $\sqrt{110.25}$ is between 10 and 11. When comparing with tenths, the number is actually equal to 10.5 because $10.5^2 = 110.25$. Therefore, the length of the hypotenuse is 10.5 inches.

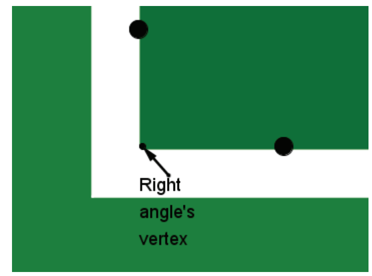
The perimeter of the triangle is
6.3 in. + 8.4 in. + 10.5 in. = 25.2 in.

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Draw a right triangle and label two of the sides with their lengths. Ask your child to use the Pythagorean theorem to approximate the unknown length of the third side to the nearest tenth or hundredth.
- Challenge your child to use the converse of the Pythagorean theorem to confirm right triangles in your surroundings. Instruct your child to mark two points on the legs of a right triangle (e.g., the sides that form the corner of a tennis court, as shown in the image at the right). Next, ask her to measure from the vertex of the right angle to each point she marked and then to measure diagonally from point to point. If the angle measures 90 degrees, your child can substitute the measurements she found for the variables in the Pythagorean theorem to yield a true statement, such as $6^2 + 8^2 = 10^2$ or, in other words, $100 = 100$. (Depending on the precision of your child's measurements and the accuracy of her measuring tool, it is likely that $a^2 + b^2$ will be only approximately equal to c^2.)



EUREKA MATH™ TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

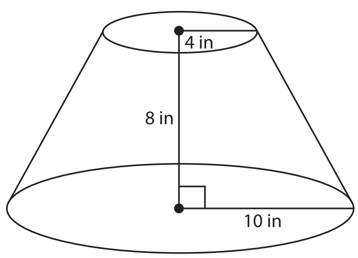
In this topic, students use the Pythagorean theorem to determine an unknown dimension (e.g., radius, **lateral length/slant height**) of a cone or a sphere, given the length of a **chord**, and to find the volume or surface area of a figure by using that dimension. Students are introduced to **truncated cones** and calculate their volumes. Students also learn that the volume of a pyramid is exactly one-third the volume of a rectangular prism with the same base area and height. In addition, students determine the volumes of **composite solids** composed of cylinders, cones, and spheres. Students then apply their knowledge of volume to compute the average rate of change in the height of the water level when water drains into a cone-shaped container. The lessons in this topic challenge students to reason while making sense of problems. Students apply their knowledge of concepts they have learned throughout the year to persevere in solving problems.

You can expect to see homework that asks your child to do the following:

- Use the Pythagorean theorem to find unknown lengths of segments in three-dimensional figures.
- Find the volume and surface area of a variety of solids, including composite solids.
- Use similar triangles to find unknown lengths of segments in pyramids and truncated cones.
- Using knowledge of volume, determine the number of minutes it would take to fill a particular three-dimensional figure.

SAMPLE PROBLEM *(From Lesson 20)*

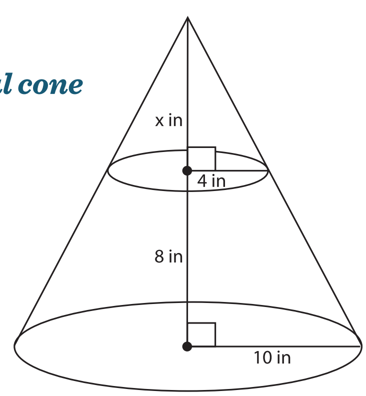
Determine the volume of the truncated cone shown below.



Let x inches represent the height of the small cone on top of the truncated cone.

$$\frac{4}{10} = \frac{x}{x+8}$$
$$4(x+8) = 10x$$
$$4x + 32 = 10x$$
$$32 = 6x$$
$$\frac{32}{6} = \frac{6x}{6}$$
$$5.\overline{3} = x$$

We must use the original cone to determine the height.



The volume of the whole cone is approximately $\frac{1}{3}\pi(10)^2(13.3)\text{in}^3$ ***or*** $\frac{1330}{3}\pi\ \text{in}^3$.

SAMPLE PROBLEM *(continued)*

The volume of the top small cone is approximately $\frac{1}{3}\pi(4)^2(5.3)\text{in}^3$ ***or*** $\frac{84.8}{3}\pi\ \text{in}^3$.

The volume of the truncated cone is equal to the volume of the larger cone minus the volume of the smaller cone. $\frac{1330}{3}\pi\ \text{in}^3 - \frac{84.8}{3}\pi\ \text{in}^3 = \frac{1245.2}{3}\pi\ \text{in}^3$

Therefore, the volume of the truncated cone is approximately $\frac{1245.2}{3}\pi\ \text{in}^3$.

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Review the formulas for volume with your child. (See Module 5 Topic B.) Students will use these formulas in application problems throughout this topic.
- With your child, examine objects in your home or outdoors. Talk about which solids make up each object. For example, a game piece in a board game may be composed of a sphere on top, a cylinder in the middle, and a cone as the base. Practicing this method of visual decomposition will help your child find the volumes of composite solids.

TERMS

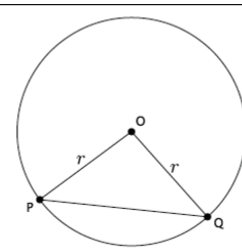


Figure 1

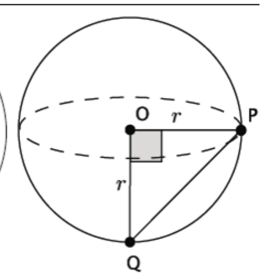


Figure 2

Chord: A straight line segment connecting two points on the outer edge of a circle or sphere. In Figures 1 and 2, $\overline{PQ}$ is a chord.

Composite solid: A solid figure made up of two or more solid figures. For example, a sharpened pencil can be viewed as a cylinder (the unsharpened portion) combined with a cone (the sharpened portion).

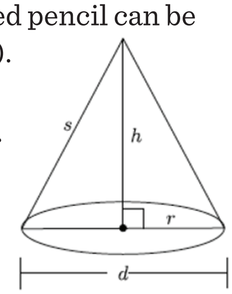


Figure 3

Lateral length/Slant height: The length of the shortest segment that connects the vertex of a pyramid or cone to a point on the edge of its base. In Figure 3, the slant height is represented by the variable s.

Truncated cone: The part of a cone that remains after a portion of the top of the cone has been removed. (See Sample Problem.)